Q1. [7 points] The truth table for NAND operation ($\uparrow$) is defined as

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

Use truth table to show whether the following is true or false.

$$\underbrace{p \wedge (q \vee r)}_{①} \equiv (p \uparrow (q \vee r)) \uparrow \underbrace{(p \uparrow (q \vee r))}_{②}$$

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p	q	r	$q \vee r$	$p \wedge ①$	$p \uparrow (q \vee r)$	$② \uparrow ②$
T	T	T	T	T	f	T
T	T	f	T	T	f ✓	T ✓
T	f	T	T	T ✓	f	T
T	f	f	f	f	T	f
f	T	T	T	f	T	f
f	T	f	T	f	T	f
f	f	T	T	f	T	f
f	f	f	f	f	T	f

$\equiv$

Q2. [8 points] Let p, q, and r be the propositions defined as

p : "I am thirsty".
q : "My glass is empty".
r : "It is three o'clock".

Write the following statements in symbolic form.

(a) I am thirsty and my glass is not empty if it is three o'clock.

$r \to (p \wedge \neg q)$ ✓

(b) It is three o'clock whenever I am thirsty.

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$p \to r$ ✓

(c) It is not the case that it is three o'clock and my glass is empty.

$\neg(r \wedge q)$ ✓

(d) My glass is empty unless it is three o'clock and I am not thirsty.

$\neg(r \wedge \neg p) \to q$ ✓

Q3. [8 points] Let

$E(x)$: "x is an even integer"
$P(x)$: "$x > 0$"
$D(x, y)$: "y is divisible by x"

Assume the domain is the set of integers $\mathbb{Z}$, write each of the following in formal symbolic form using *only* the above predicates and appropriate quantifiers.

(a) Every positive and even integer is divisible by 2.

$\forall x \in \mathbb{Z}; (E(x) \land P(x)) \to D(2,x)$

(b) Some none-positive odd integers are not divisible by 2.

$\exists x \in \mathbb{Z}: \neg E(x) \land \neg P(x) \land \neg D(2,x)$

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(c) No positive integer divisible by 9 is divisible by 11.

$\forall x \in \mathbb{Z}: (P(x) \land D(9,x)) \to \neg D(11,x)$

(d) The successor of every even integer is odd.

$\forall x \in \mathbb{Z}: E(x+1) \to \neg E(x)$

$\equiv E(x) \to \neg E(x+1)$

Q4. [4 points] A Mobile Phone shop displays the sign "*Good mobile phone is not heavy*", and a competing shop displays the sign "*Mobile phone not heavy is good*".

(1) [2 pts.] Let g be "Mobile phone is good" and h be "Mobile phone is heavy". Convert the above two signs into symbolic form.

① $g \to \neg h$ ② $\neg h \to g$
$\equiv \neg g \lor \neg h$ $\equiv h \lor g$

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(2) [2 pt.] Are the two signs equivalent? Justify your answer.

No because it is equivalent only when the second one is it's contrapositive. ~~contrapositive~~ converse OR show by truth table.

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Also, $\neg g \lor \neg h \not\equiv h \lor g$,

Q5. [8 points] Show that the following argument is valid.

1. $p \wedge \neg r$ $\equiv p \wedge \neg r$
2. $q \to r$ $\equiv \neg q \vee r$
3. $p \wedge s \to n$ $\equiv \neg p \vee \neg s \vee n$
4. $\neg w \vee \neg s \to q$ $\equiv \neg p \vee \neg s \vee n$

$\therefore n \wedge w$ $\equiv (w \wedge s) \vee q \equiv (q \vee w) \wedge (q \vee s)$

(1) $\neg q \vee r$, $\neg r$, $\therefore \neg q$

(2) $\neg p \vee \neg s \vee n$, p, $\therefore \neg s \vee n$

(3) $q \vee w$, $\neg q$, $\therefore w$

(4) $\neg s \vee n$, $q \vee s$, $\therefore q \vee n$

(5) $q \vee n$, $\neg q$, $\therefore n$

$\boxed{\therefore n \wedge w}$ valid.

(1) $\neg q \vee r$, $\neg r$, $\therefore \neg q$

(2) $\neg p \vee \neg s \vee n$, p, $\therefore \neg s \vee n$

(3) $q \vee w$, $\neg q$, $\therefore w$

(4) $\neg s \vee n$, $q \vee s$, $\therefore n \vee q$

(5) $q \vee n$, $\neg q$, $\therefore n$

(6) w, n, $\boxed{\therefore n \wedge w}$

$\therefore$ Valid

show the steps.

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